Using Thought Experiments to Teach Einstein's Ideas

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ABSTRACT. The main concepts of quantum theory and relativity are generally only hinted at in senior high school physics. We begin by addressing the main historical antecedents of Einstein's work, namely, two thought experiments (TEs) by Galileo that were foundational for his special theory of relativity (STR). We then illustrate the use of TEs that led to the conclusions reached in Einstein's article of 1905, in what is now called the special theory of relativity (STR). We conclude with the TE that touches on the foundation of the general theory of relativity (GTR). We argue that the TEs presented can be injected into the senior high school or first year college physics program, as the instructor sees fit, with the following in mind: the story aspect, or the historical background of the TE, must be considered a necessary but not sufficient condition for a successful presentation of the TEs. Finally, we recommend that the TEs must be discussed in such a way that the argument and the quantitative aspect of the TE is comprehensible to the student. We trust that to achieve that goal we have given the teacher valuable suggestions.

Introduction

The main concepts of quantum theory and relativity are generally only hinted at in senior high school physics. It is certain that during and long after 2005, the centenary of the *annus mirabilis* of Einstein's revolutionary papers, we will again see a call for an earlier introduction of modern physics into the secondary physics curriculum, especially of the ideas of Einstein. In this paper, we are advocating the use of thought experiments (TEs) as a means to introduce modern topics, in particular Einstein's work on relativity. We begin by addressing the main historical antecedents of his work, namely two thought experiments by Galileo, that were foundational for his special theory of relativity (STR). We will then illustrate the use of TEs that led to the conclusions reached in his article of 1905, in what is now called the special theory of relativity. We will conclude with the TE that touches on the foundation of the general theory of relativity (GTR).

Thought Experiments

We begin by presenting a set of TEs that we believe address the fundamental concepts leading to Einstein's theory of relativity. We are aware of the fact that most of these TEs are readily available from diverse sources. However, as far as we know, there are very few publications that discuss explicitly, from a historical perspective, the connections between TEs and the placing of the STR and the GTR into the secondary physics curriculum. To enrich the presentation we have tried to embed relevant historical comments to show the evolution of ideas and concepts that led to Einstein's work. This summary should therefore assist the physics teacher in preparing her lessons for the presentation of the TEs as well as becoming more acquainted with the history connected with them. Finally, we should remind our students that the process of argumentation given in the TE is as important, and often more important than the final conclusion.

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GALILEO'S THOUGHT EXPERIMENTS

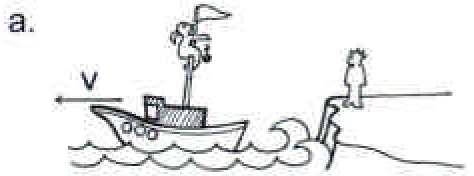
Galileo argued that if a sailor dropped a canon ball from the mast of a ship that was smoothly moving with a constant velocity, the cannon ball would fall directly below him. Whatever the sailor would do, assuming a smooth constant velocity in a straight line, he would not be able to tell that the ship was in motion if he confined his attention to the ship. Using modern terminology, Galileo thus claimed that *there is no distinction between constant motion and rest.* All of us have experienced this when traveling in train on a level track moving with a constant speed.

Students have been told that all inertial frames are equivalent, a principle that we will find Einstein using as the basis of one of his two axioms for his special theory of relativity. We should remember, however, that in 1905 there existed no evidence against the general validity of Galilean invariance in pure mechanics (Pais, 1982, p. 140). The transformation equations **$x' = x - vt$; $y' = y$, $z' = z$, $t' = t$, and $v = v_1 + v_2$** (velocity addition) were considered valid for all motion.

Galileo's most famous TE, however, is the one that shows that all objects, regardless of their weight, must fall at the same rate. He argued that, according Aristotle, heavier bodies fall faster than light ones (H , L). But what would happen if a heavy cannon ball were attached to a light musket ball? Reasoning in the manner of Aristotle leads to an absurd conclusion. First, the light ball will slow down the heavy ball (acting as a kind of drag), so the speed of the combined system would be slower than the speed of the heavy ball falling alone ($H > H+L$). But the combined system is heavier than the heavy ball alone, so it should fall faster ($H + L < H$). But that is absurd, for how can the heavy ball be both faster and slower than even the combined system? Thus Galileo refutes Aristotle's theory of free fall.

This TE shows the equivalence of inertial mass and gravitational mass and later becomes fundamental in Einstein's GTR.

Figure 1a: Galileo's ship TE.



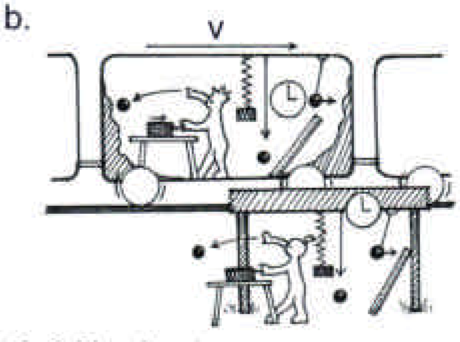


Figure 1b: Galilean Invariance

Figure 2: Galileo's Falling Weights TE

THOUGHT EXPERIMENTS TO INTRODUCE EINSEIN'S IDEAS

Already as an adolescent, Einstein pondered the foundations of physics. In 1895, when he was 16 years old, he sent a paper to his uncle in Belgium about the ether and the magnetic field (Pais, 1982, p. 130). In this paper Einstein describes his TE in which riding a light wave one would see an electromagnetic field at rest, a paradox according to Maxwell's laws.

Einstein knew that the Newtonian concept of time, together with the concept of absolute space as presented in his *Principia*, is deeply enshrined in our common sense world: Newton's view of time as an absolute idea is one of the fundamental assumptions of Newtonian physics. According to Pais (1982), Einstein felt certain of the truth of the Maxwell-Lorentz equations of electrodynamics and he reasoned that the invariance of the speed of light would ensure that these relations should hold in all inertial frames of reference.

However, this invariance was in conflict with the rule of addition of velocities (see above) according to the conventional addition of velocities based on Galilean relativity. Einstein puzzled over this problem for almost a year until he was convinced that *there was an inseparable connection between time and signal velocity*. Before he was comfortable in using the idea of different times for different inertial frames of reference, Einstein used a TE to give the idea of simultaneity an operational definition. To show that two events which are simultaneous in one frame of reference are not simultaneous in another he argued along the line found in the following TE, which is a "textbook" version of the TE that Einstein described in his paper. It is interesting to note that the railroad car version is actually a TE Einstein first described in a popular book he wrote in 1917 (See references). Many versions of this TE can be found in text books. The following version is a composite of several sources:

> As a train passes the physics student, two lightening bolts strike either end of the train leaving burn marks on the track and sending flashes of light into the train. The passenger sees the lightning bolts strike the two ends of the train simultaneously. The physics student sees the two flashes of light meet at the center of the distance between the two burn marks, (as she must, since the speed of light in all directions is the same and independent of the source or observer.)
>
> The passenger, however, sees things differently. He is standing in the middle of the train. After the lightning strikes he first sees a flash of light from the front of the train then a bit later a flash of light from the back of the train. As the speed of light is a constant in his frame as well she must conclude that the front of the train was stuck by lightning before the back of the train.

The physics student and the passenger disagree as to whether the two events (the front and back lightning strikes) are simultaneous. But they are both correct. One might be tempted to say that the physics student must be correct since the passenger is moving and she is not. But the first postulate tells us that we can not tell the difference between moving frames. It follows that events which are simultaneous in one inertial frame, are not simultaneous as seen in another inertial frame. This means that, if the principle of the constancy of light is accepted, then the conclusion that *events that are simultaneous in one frame of reference may not be simultaneous in another* necessarily follows. See Fig. 3.

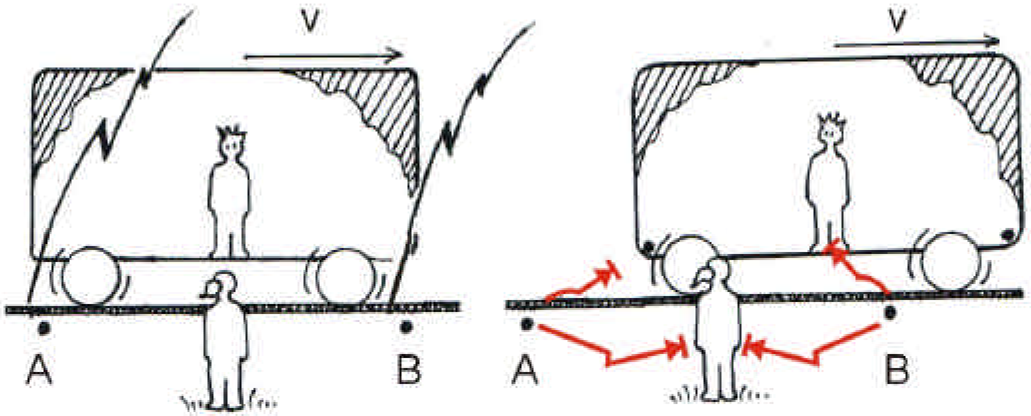


Figure 3: Einstein’s TE about the relativity of simultaneity.

The following is a TE that we find in many college textbooks. It seems that this TE first appeared in the 1960's and was instantly recognized as an excellent pedagogical tool for teaching the ideas of the STR in first year university classes. In fact, when one first encounters it one is tempted to think that it is just another TE devised by Einstein himself.

What follows is the time dilation effect TE, based on a standard physics textbook that we thought was especially well presented (Serway and Faughn, 1999). This TE is really an extension of the previous TE where we analyze the situation and derive a relationship between the measurement of time in the two frames of reference, the observer on the ground and the observer in the car.

Consider a vehicle moving to the right with a speed v, as shown in Fig.7. A mirror is fixed to the ceiling of the vehicle, and an observer O’ at rest in this system holds a flashgun at a distance d from the mirror. At some instant the flashgun goes off and a pulse of light is released. Because the light pulse has a speed c , the time it takes to travel from the observer to the mirror and back again is found from the definition of velocity, D**t’ = distance / velocity = 2d / c** where D**t’** is the time interval as measured by O’, the observer who is at rest in the moving vehicle.

We will consider the same set of events as viewed by an observer in O, in the stationary frame. According to this observer, the mirror and flashgun are moving to the right with a speed of v. The sequence of events now is different, as seen in this frame. By the time the flashgun reaches the mirror, the mirror will have moved a distance of **v** D**t / 2**, where D**t** is the time it takes the light pulse to travel from O’ to the mirror and back, as measured by the stationary observer. Comparing (a), (b), and (c) in Fig. 7. it is clear that the light must travel farther in the stationary frame than in the moving frame.

We can now use Einstein’s second postulate, namely, that the speed of light must be c as measured by both observers. It follows then that the time interval D**t**, *as measured by the observer in the stationary frame,* is longer than the time interval D**t’,** as measured by an observer in the moving frame. We are now able to obtain a relationship between D**t** and D**t'**, by using Pythagoras' theorem applied to the right angled triangle.

$(c\,Dt/2)^2 = (v\,Dt/2)^2 + d^2$
Solving for D**t** gives $Dt = 2d/c(1-v^2/c^2)^{1/2}$
Since $Dt = 2d/c$
It follows that $Dt = Dt'/(1-v^2/c^2)^{1/2} = g\,Dt'$
Where $g = 1/(1-v^2/c^2)^{1/2}$

This says that "the time interval, as measured by the observer in the stationary frame, is longer than that measured by the observer in the moving frame" by a factor of g.

The general conclusion drawn from this analysis is that "all physical processes, including chemical and biological reactions, slow down *relative to a stationary clock* when they occur in a moving frame".

The time dilation relationship immediately raised the question: Would the observer in the moving frame not come to the same conclusion about the "stationary" frame as far is time dilation is concerned? The answer is yes, and this leads us to our next thought experiment.

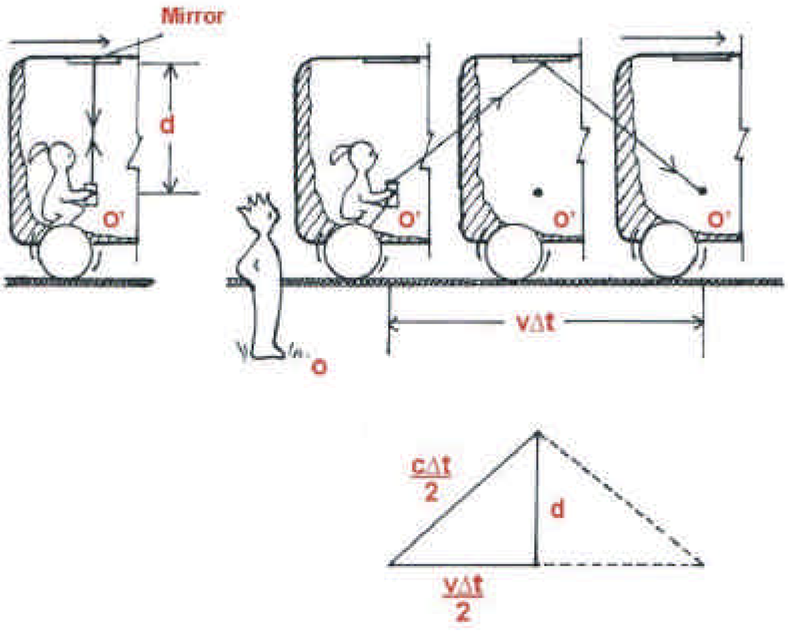


Figure 4: A TE to show the time dilation effect

THE TWIN PARADOX

The oldest and best known of all relativistic paradoxes, is the so called *twin paradox*. This paradox was the testing ground for the counterintuitive results of time dilation that we have already discussed. Physicists and students seldom read Einstein's original papers, and therefore their understanding is based only on textbooks. Unfortunately, most textbooks devote only a few short paragraphs to the paradox and resolve it quickly, not indicating any reservations about this conventional discussion.

Einstein makes a brief reference in his 1905 paper to what he called a "peculier consequence", and referred to it as a *theorem*, but it was dubbed the *clock paradox* by the French physicist P. Langevin in 1911. Ultimately, it became the celebrated *twin paradox* that dealt with the so-called asymmetrical aging problem that was hotly debated in the journal *Nature* in the years 1957-1959 (French, 1970, Schlegel, 1951), and is still debated today. The *twin paradox* was originally raised in an attempt to use symmetry and time dilation to show that the STR was inconsistent.

Here is the paradox: Barb travels in a straight line at a relativistic speed v to some distant location. She then decelerates and returns. Her twin brother Bob stays at home on Earth. The situation is shown in the diagram, which is not to scale. See Fig. 5.

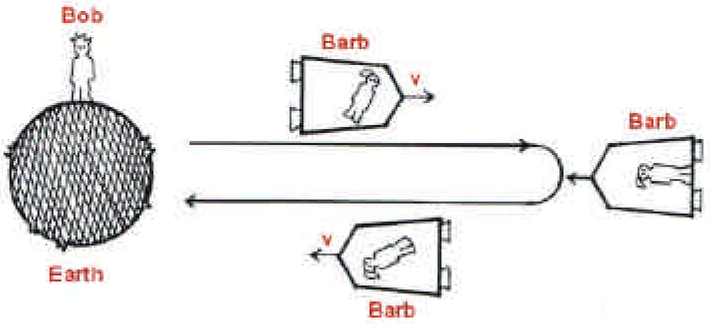


Figure 5: The Twin Paradox

Bob observes that Barb's on-board clocks (including her biological one), which run at Barb's proper time, 'run slow' on both the outbound and return leg. He therefore concludes that she will be younger than he will be when she returns. On the outward leg Barb also observes Bob's clock to 'run slow', and she observes that it ticks slow on the return run. So, will Barb conclude that Bob will have aged less? And if she does, who is correct? According to the proponents of the paradox, there is a symmetry between the two observers, so, using relativity, each will predict that the other is younger. This cannot be simultaneously true for both. Therefore, if the argument is correct, relativity is wrong.

Most discussions of the twin paradox in textbooks point out that the motion of the twins is not symmetrical because there is an acceleration and deceleration at both ends of Barb's journey. Thus, Barb finds herself in a non-inertial reference frame four times during the trip while Joe has stayed in an inertial frame of reference all the time,. According to STR, we simply refer to the clocks which are at all times in a single inertial system. Therefore, only Bob is allowed to apply the time dilation formula to Barb's entire voyage and the correct conclusion is that Barb is younger on her return. The asymmetric aging of the twins is then considered a consequence of the laws of nature, just as the more familiar world of our everyday experience is. This is just the way nature works. Unfortunately, most textbooks imply that this is an obvious choice and do not offer further discussion.

However, the twin paradox almost immediately inspired significant debate. Shortly after Einstein's paper appeared, P. Longevin appealed to the acceleration phase of the journey as being significant in deciding who would be the younger twin at the end of the journey. Longevin's analysis was widely read and it challenged Einstein's own explanation based on the STR alone. Later, in 1913 , Max von Laue returned to the acceleration matter, noting that "we can make the times of uniform motion arbitrarily greater than those of the acceleration". Additionally, Lorentz in his lectures at Leiden, also emphasized Einstein's simultaneity argument as sufficient.

It is interesting that Einstein in 1918 invoked his newly completed and later (1919) confirmed GTR to explain the paradox. Pesic (2003) believes Einstein's emphasis on the GTR ensured that the argument that only the GTR is able to explain the paradox prevailed. We now know that the STR, using the simultaneity argument, is able to resolve the twin paradox (French, 1968). It is interesting to note that, according to the physicist Richard Schlegel, Einstein said to him in 1952 that "general relativity has nothing to do with the clock paradox". There is still confusion in the literature about how the STR and the GTR relate to the twin paradox.

EINSTEIN RE-EXAMINES A COMMON-PLACE PHENOMENON

It is intriguing that it was asymmetries that initially inspired Einstein to formulate many of his ideas around STR. In fact, Einstein's famous paper on relativity (actually titled: "On the electrodynamics of moving bodies") begins this way:

> It is known that Maxwell's electrodynamics--as usually understood at the present time--when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena.

Einstein wonders how it was possible to distinguish between the motion of a conductor towards a magnet on the one hand, and the motion of a magnet towards the conductor on the other? Maxwell's theory clearly distinguished between these two, as seen in this common-place phenomenon. Einstein goes on to describe the reciprocal electrodynamic action of a magnet and a conductor. He noted that the "observable phenomenon here depends only on the relative motion of the conductor and the magnet".

How was it possible to distinguish between the motion of a conductor towards a magnet on the one hand, and the motion of a magnet towards the conductor on the other? Maxwell's theory clearly distinguished between these two, as seen in this common-place phenomenon.

Physicists are generally aware that relativity is involved in this commonplace demonstration (we can refer to it as the 'Faraday magnet and coil induction experiment'). It is often shown to students as early as in junior high school. Unfortunately, when it is again discussed in senior high school, it is often repeated with only a quick reference to Faraday's law. However, teachers generally do not exploit the full significance of it as a simple demonstration of relativity. One reason may be that physics teachers in general are not aware that this simple experiment demonstrates a truly relativistic effect at very low velocities.

First of all, it shows that physical results depend only on the relative motion, and that electric and magnetic fields manifest themselves differently to moving observers. Secondly, a discussion of this demonstration can provide the motivation to study relativity the same way as it motivated Einstein to go beyond the conventional explanation. This is a good example that illustrates the notion that we "see" with our ideas rather than our eyes. Refer to Fig. 6 for detail.

Case A: The magnet is held stationary and the coil is moved toward the magnet. Here the force that moves the electrons around the coil to produce the deflection in the galvanometer is the Lorentz force given by **F = q v B / c**, where v is the velocity of the coil toward the magnet.

Case B: The coil is held stationary and the magnet is moved toward the coil, In this case the effect of the magnetic field does not effect the electrons in the coil, since their velocity is initially zero. This time, however, what produces the effect in the galvanometer is Faraday's law of induction, namely that a changing magnetic field produces a circular electric field:

D**B** / D**t** = **E** (actually **Grad E = 1/c dB/dt**).

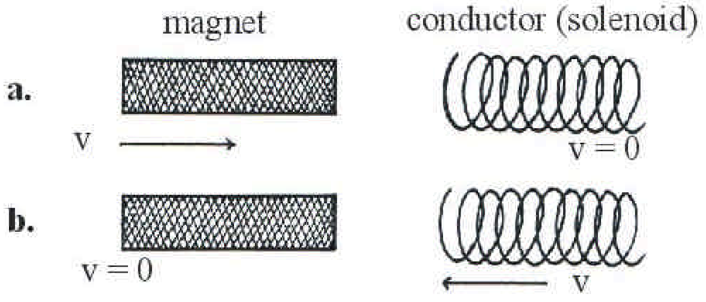


Figure 6: Magnet and Solenoid

It can now be stressed that

1. In case A there is only a magnetic field in the classroom, but in case B there is also an electric field in the classroom.

2. The galvanometer deflection is truly a relativistic effect, proportional to v.

EINSTEIN'S ELEVATOR TE: A POSTSCRIPT

In conclusion**,** we must make a brief connection with the GTR. Shortly after the publication of his STR paper in 1905, Einstein wondered how relativity theory could embrace all frames of reference**,** including accelerating frames of reference. Einstein realized that his STR was not compatible with Newton's law of gravitation.

Einstein noted when the mass-energy equivalence is combined with the gravitational equivalence principle, it becomes technically incompatible with the STR. He therefore tried to generalize the STR to accommodate gravitation consistently.

In November of 1907, while working on a paper on the SRT, Einstein tried to modify Newtonian theory of gravitation "in such a way that its laws would fit in the special relativity theory" (Pais, p. 178). Later, in his Kyoto lecture of 1921 Einstein recalls:

> Then there occurred to me the 'happiest thought of my life' in the following form. The gravitational field has only a relative existence in a way similar to the electric field generated by electromagnetic induction. Because for an observer falling freely from the roof of a house there exists - at least in his immediate surroundings - no gravitational field. Indeed, if the observer drops some bodies then they remain relative to him in a state of rest or in uniform motion. The observer

> therefore has the right to interpret his state as ‘at rest’. Because of this idea, the uncommonly peculiar experimental law that in a gravitational field all bodies fall with the same acceleration, attained at once a deep meaning.

Einstein then concludes by saying that the experimentally known fact that all masses fall with the same acceleration in a gravitational field is a powerful argument for extending the relativity postulate to coordinate systems which, relative to each other, are in non-uniform motion.

We will conclude with Einstein’s elevator TE that first appeared *in The Evolution of Physics*, a book that was written with Leopold Infeld for the general public in 1938.

We have come full circle with our TE presentation. We began with Galileo and his ship in part one that represented an inertial frame of reference and now we are looking at the question of how accelerating frames of reference and gravity are related. To illustrate that principle, Einstein proposed a thought experiment involving elevators, with no windows. In his original version, however, he placed the non-accelerating elevator on the Earth, the accelerating elevator just above it. We will change this TE a little (with apologies to Einstein) and place one elevator in deep space (no gravity) and the other on the Earth at rest. See Fig. 9.

One elevator is in deep space and is moving with an acceleration of 1 g . An identical elevator is placed on the surface of the earth, but is at rest. Einstein claimed that there is no experiment that can be done in the elevators to decide in which elevator the physicist is doing the experiments.

If mechanical experiments cannot distinguish between the two elevators, an optical experiment might. Using a laser we would find (assuming that the slight change were measurable) that the trajectory of the laser would follow parabolic path in both elevators, in analogy with the parabolic path of a ball thrown in a horizontal direction in the gravitational field of the earth. So light, according to Einstein, should bend in a gravitational field in a predictable way. Finally, light always follows a shortest-time path between two points, so the shortest distance between two points will no longer be a straight line but a *geodesic* that describes the path of light (Stinner, 1995).

These TEs then convinced Einstein that “the correct way to formulate a theory of gravity was to construct a theory of motion that was consistent with the STR which gave, in the non-relativistic limit, Newton’s inverse square law of gravity” (Ryder, 1987, p. 348). Finally, students should compare Galilean relativity with Einsteinian by going back to our first TE.

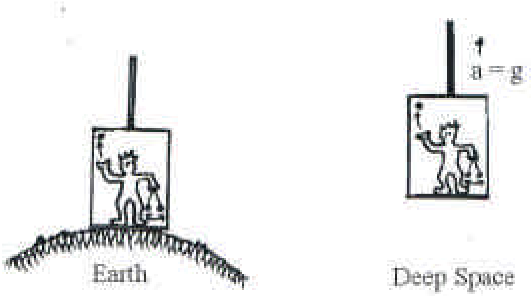


Figure 7: Einstein’s elevator TE

As described in the elevator TE, Einstein concluded that the effects of gravitation and those due to acceleration cannot be distinguished. Newtonian mechanics distinguishes between the motion of a body that is inertial (subject to no forces) and the motion of a body subject to the action of gravity. The former is rectilinear and uniform in an inertial system; the latter occurs in curvilinear paths and is non uniform (Born, p. 315). The principle of equivalence, however, does not allow this distinction. Einstein's mandate now was to state the law of inertial motion in the generalized sense. The solution of this problem banishes both the notion of absolute space and force.

We will make only a few comments here and refer the reader to two excellent sources for a detailed discussion of these. For the STR and its experimental consequences we recommend Pais (1982) and for the GTR we found Rowlands (1987) to be an outstanding paper.

First of all, as far as the STR is concerned, we should keep in mind that the three quantities, time dilation, length contraction, and apparent mass increase are intrinsically connected; *therefore one effect necessarily implies the other*. In addition, students should be told that the experimental tests to confirm these took a long time and the findings were often controversial. For example, it was not until about 1910 that experiments with cathode rays clearly showed that the Einsteinian interpretation was superior to any of the proposed classical hypotheses to explain the observed mass increase of the electrons moving with a high velocity. Time dilation (and therefore also length contraction) was not confirmed by an experiment until 1941, detecting the arrival of muons produced in the atmosphere by cosmic rays. The famous equation $E = mc^2$ was confirmed in the Compton Effect in 1923, and later by the neutron-induced fission reaction first reported by Otto Hahn and Fritz Srassmann in 1938. About the same time Hans Bethe presented a model for the thermonuclear reactions in the sun that showed that the age of the sun must be about 4-5 billion years.

Secondly, as far as the GTR is concerned, all three predictions made by Einstein in 1915 have been confirmed. The gravitational redshift is the simplest of the effects to explain but the most difficult to test. In fact, it was not confirmed until 1960, five years after Einstein's death. The gravitational deflection of light from a star, as it passed close to the sun, was measured in 1919 by Eddington and his group during a solar eclipse, confirming Einstein's prediction. This is the highly publicized observation that launched Einstein to instant world fame.

The last effect, the precession of Mercury's perihelion, was the first to be confirmed by the GTR. Einstein knew that in 1859 the French astronomer Leverrier found, later corroborated by the British astronomer Simon Newcombe, that of the 574 arc seconds per century, 531 could be accounted for by perturbations due to other planets. But the remaining 43 seconds resisted all analysis (Rolands, 1987, p. 54). Einstein found that his GTR could account for these 43 arcsecond exactly. This incredible match between theory and observation delighted Einstein and he wrote: 'I was beside myself with ecstasy for days'.

Finally, two things should be mentioned that are generally not known. First, Einstein predicted the gravitational redshift effect as early as 1907 and derived the equation based on more rigorous arguments in 1911. Clearly, one does not need the GTR to derive this equation. Secondly, it seems that all three effects mentioned in connection with the GTR can be derived, using only Newtonian theory and the STR (Rowlands, 1987). This fact is useful to initiate discussion about the GTR and how it relates to Newton's theory of gravity and the STR.

Conclusion

The TEs presented here can be injected into the senior high school or first year college physics program as the instructor sees fit, with the following in mind: the story aspect, or the historical background of the TE, must be considered a necessary but not sufficient condition for a successful presentation of the TE. The TE must also be discussed in such a way that the argument and the quantitative aspect of the TE is comprehensible to the student.

For the STR the teacher should actually do the solenoid-magnet demonstration, and then discuss the implications as outlined above. For the GTR you can demonstrate the equivalence of gravitational and accelerated frames of reference. One way to do this is to use the old demonstration of a cork floating in a bottle filled with water that is often used to demonstrate "centripetal" acceleration, or attach a helium balloon to the bottm of a car and find out what happens to the balloon. Putting it anthropomorphically, the cork and the balloon cannot differentiate between acceleration and gravity.

We should be reminded that physics teachers are still confronted by the prevailing myth that Einstein's relativity theories are understood by only a few physicists. Countering this claim, Abraham Pais, referring to Einstein's great paper on relativity, challenged physics teachers: "It also seems to me that this kinematics, including the addition of velocity theorem, could and should be taught in high schools as the simplest example of the ways in which modern physics goes beyond intuition". (Pais, 1982, p. 141).

At the same time, however, the remark made by a noted expert in the field, A.P. French, rings true for all of us who have wrestled with the concepts and assumptions of the STR over many years:

> No matter how long one has lived with the results of special relativity, there is something very counterintuitive about it. (French, 1968, p. 89).

It seems then that early and long exposure to these concepts and ideas is necessary in order to become comfortable with them. As Kuhn pointed out in his *Structure of Scientific Revolutions,* those who come to revolutionary new ideas (such as the ones in the STR) late in life can sometimes be *converted*, while those who grow up with them are *persuaded* at an early age and become comfortable with theses ideas and concepts. Introducing students early to the ideas of Einstein in what Jerome Bruner called "an intellectually honest way", will ensure a much longer gestation time to prepare them for a less jolting encounter of these ideas later.

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